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ON THE STABILITY OF LAMINAR PLUMES: SOME NUMERICAL SOLUTIONS AND EXPERIMENTS

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Abstract—An investigation of the hydrodynamic stability of a laminar plume arising from a horizontal line source of heat was carried out using the Tollmien–Schlichting theory of small disturbances. Inviscid solutions of the Orr–Sommerfeld equation were obtained for both symmetric and asymmetric disturbances and the effect of the Prandtl number on the inviscid stability was calculated for asymmetric disturbances. The base flow was found to be less stable for the asymmetric mode. In addition, the full disturbance momentum equations, coupled and uncoupled from the energy equation, were numerically integrated with the boundary conditions appropriate for asymmetric disturbances superimposed on the symmetric plume base flow. Neutral stability curves have been obtained in terms of the Grashof number.

The predominance of the assumed asymmetric mode of flow oscillation was verified experimentally by perturbing a plume, in air, with a vibrating ribbon. A Mach–Zehnder interferometer was used to observe the disturbances as they were convected downstream.

The experimental results demonstrate that sufficiently high frequency disturbances are stable as they are convected downstream.

NOMENCLATURE

F', dimensionless velocity of the base flow, $F' = u/U^*$;

f, disturbance frequency;

G, modified Grashof number, $G = 2(\sqrt{2})(Gr)^{\frac{1}{4}}$;

Gr, Local Grashof number,

$$Gr \equiv \frac{g\beta^* x^3(T_0 - T_\infty)}{\nu^2};$$

g, acceleration of gravity;

k, thermal conductivity of the fluid;

s, dimensionless disturbance temperature amplitude function, $s \equiv \bar{s}/(T_0 - T_\infty)$;

T, base flow temperature;

t, instantaneous flow temperature;

U^*, characteristic velocity, $U^* \equiv \nu G^2/4x$;

u, instantaneous velocity in vertical direction;

x, coordinate in vertical direction;

y, coordinate in horizontal direction;

α_{re}, dimensionless wave number, $\alpha = \bar{\alpha}\delta$;

α_{im}, amplification factor along x coordinate;

β_{re}, dimensionless frequency of disturbance, $\beta = \bar{\beta}(\delta/U^*)$;

β_{im}, time amplification factor at given x;

β^*, coefficient of volumetric expansion;

δ, characteristic length, $\delta \equiv 4x/G$;

ϕ, dimensionless disturbance velocity amplitude function, $\phi = \bar{\phi}/\delta U^*$;

Φ, dimensionless temperature of the base flow,

$$\Phi \equiv \frac{T - T_\infty}{T_0 - T_\infty};$$

η, dimensionless distance in y direction, $\eta = y/\delta$;

λ, wavelength of the disturbance;

ρ, density of the base flow;

ν, kinematic viscosity of the fluid;

σ, Prandtl number;

τ, dimensionless time, $\tau = \bar{\tau}/\delta/U^*$;

μ, dynamic viscosity of the fluid.

Other symbols

$'$, differentiation with respect to the similarity variable η;

$(-)$, dimensional quantity;

$(\sim)$, disturbance quantity.

INTRODUCTION

THE TRANSITION from laminar to turbulent flow in a fluid has drawn the attention of many investigators throughout the years. The analysis of stability is commonly done by perturbing a given steady-state solution of the equations of motion with small periodic velocity and/or temperature disturbances. Usually a single sinusoidal mode is considered. If the disturbance decays, the flow is said to be stable. If the disturbance grows or remains at some magnitude different from zero, the flow is said to be respectively unstable or neutrally stable.

Squire (1933) showed that, in boundary layer flow, two-dimensional disturbances amplify at a lower Reynolds number than three-dimensional ones, and are, therefore, the least stable. Knowles and Gebhart [1] extended this proof to a natural convection flow where velocity and temperature disturbances are coupled. In light of these results, it is sufficient in this investigation to consider only two-dimensional disturbances superimposed upon the parallel flow.

The present paper presents an analysis of the hydrodynamic stability of a two-dimensional plume, flow generated by an infinite line source of heat. This flow circumstance has received little study in the past and the authors are not aware of any other study of the stability of such flows. Previous stability studies of similar flows are mentioned below.

Pai [2] considered the stability of a two-dimensional laminar jet flow of a compressible and incompressible fluid. He purported to show that there is no lower branch of the neutral curve. However, it was pointed out that the lower branch of the neutral curve may be so close to the axis $\alpha = 0$ that even for large Re the expansion in powers of $(\alpha Re)^{-1}$ is not valid.

Curle [3] considered the problem of the stability of a laminar jet with a steady velocity profile. The fourth derivative of the disturbance function is assumed to be significant only near the singular layer. Results confirm the existence of the lower branch of the neutral curve and show how quickly its asymptote, the $\alpha = 0$ axis, is approached.

Plapp [4] considered the stability of laminar flow near a vertical heated plate with a density-dependent body force. He was able to make actual calculations by neglecting coupling between the momentum and energy equations. The results were not in close agreement with existing experimental observations.

Ostrach and Maslen [5] considered the stability of fully developed natural convection flow between vertical plates. It was shown, that for large values of Re, the body forces affect the stability only through the base flow profiles. Both symmetrical and asymmetrical disturbances were considered.

The present paper considers various aspects of laminar stability limits for the two-dimensional plume flow. The full equations for linear stability theory for the plume base flow are shown to have an asymptotic (in local Grashof number Gr) inviscid behaviour. This stability limit is calculated for both symmetric and asymmetric disturbances and, for the latter case, for a wide range of Prandtl number, σ. Similarly, as Gr becomes large, the coupling effect is reduced and uncoupled neutral stability curves curves were obtained for various σ. Finally, it is shown that this coupling mechanism between temperature and velocity disturbances through buoyancy is much more important at lower values of Gr, i.e. near the threshold of instability. A coupled neutral curve is given, showing this effect.

These calculations were carried out for the plume base flows appropriate for the various values of σ. These flows were calculated from the following formulation given by Gebhart *et al.* [6]:

$$F''' + 2{\cdot}4\,FF'' - 0{\cdot}8\,F'^2 + \Phi = 0$$
$$\Phi'' + 2{\cdot}4\,\sigma\,(F\Phi)' = 0, \tag{1}$$

with boundary conditions:

$$F(0) = F''(0) = \Phi'(0) = 0, \qquad \Phi(0) = 1, \quad F'(\infty) \to 0, \tag{2}$$

where Φ are F are the temperature and stream functions, for the similar flow circumstance and the primes indicate differentiation with respect to the similarity variable η.

THE STABILITY EQUATIONS AND BOUNDARY CONDITIONS

The flow configuration for a plume above an infinitely long horizontal line source is shown in Fig. 1. The hydrodynamic stability of the

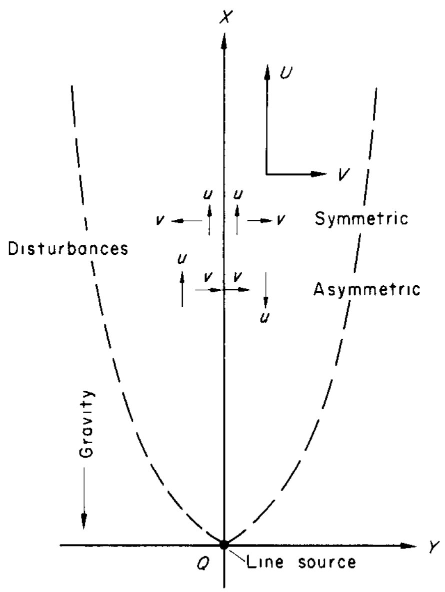

FIG. 1. Geometry.

base flow is examined by the method of small disturbances; inquiring whether a certain small disturbance, superimposed on the base flow, and satisfying the governing continuity, momentum and energy equations, is amplified or damped as it travels downstream.

The form of the disturbances stream and temperature functions assumed are:

$$\tilde{\psi}(x, y, \bar{\tau}) = \bar{\phi}(y) \exp\left[i(\bar{\alpha}x - \bar{\beta}\bar{\tau})\right] \tag{3}$$

$$\tilde{t}(x, y, \bar{\tau}) = \bar{s}(y) \exp\left[i(\bar{\alpha}x - \bar{\beta}\bar{\tau})\right] \tag{4}$$

where $\bar{\phi}$ and $\bar{s}$ are the complex amplitude functions of the disturbances. In the most general form $\bar{\beta} = \bar{\beta}_{re} + i\bar{\beta}_{im}$ where $\bar{\beta}_{re}$ denotes the frequency of the disturbances and $\bar{\beta}_{im}$ indicates how their amplitude varies with time at given x. Similarly $\bar{\alpha} = \bar{\alpha}_{re} + i\bar{\alpha}_{im}$ where $\bar{\alpha}_{re} = 2\pi/\lambda$ is the wave number of the disturbance and $\bar{\alpha}_{im}$ describes their amplification with x. In the present work $\bar{\beta}_{im}$ is taken to be zero since we are considering steady locally periodic flow. This procedure results in the same predictions of neutral stability as the more traditional measure of taking $\bar{\alpha}_{im} = 0$.

The resulting stability equations after following usual procedures of nondimensionalization are:

$$\phi'''' - 2\alpha^2\phi'' + \alpha^4\phi + s' = i\alpha G\left[\left(F' - \frac{\beta}{\alpha}\right)(\phi'' - \alpha^2\phi) - F'''\phi\right] \tag{5}$$

$$s'' - \alpha^2 s = i\alpha\sigma G\left[\left(F' - \frac{\beta}{\alpha}\right)s - \phi\Phi'\right] \tag{6}$$

where $\beta_{re}/\alpha_{re} = c$ is the wave velocity and G is an indication of the vigor of the local flow.

Equation (5) is the traditional Orr–Sommerfeld equation with an additional coupling term, s', which arises from the buoyancy effect. Combined with the disturbance energy equation, (6), the problem is of sixth-order.

The foregoing formulation is general and governs stability for any vertical natural convection boundary layer flow. The distinguishing characteristics of a particular flow and disturbance configuration are manifested only in the specification of the various boundary conditions relevant to that flow. The nature of a plume flow permits the formulation of boundary conditions at a large distance out from the plume midplane:

$$\phi'(\pm\infty) \to 0, \qquad \phi(\pm\infty) \to 0, \qquad s(\pm\infty) \to 0. \tag{7}$$

Since any solution of (5) and (6) can be considered to be a linear combination of symmetrical and asymmetrical solutions, these two extreme situations were postulated. The two types of motions were considered separately and each yields the necessary additional three boundary conditions.

$$\phi(0) = \phi''(0) = s'(0) = 0 \quad \text{(Symmetric disturbances)} \qquad (8)$$

$$\phi'(0) = \phi'''(0) = s(0) = 0 \quad \text{(Asymmetric disturbances).} \qquad (9)$$

Equations (5) and (6) with (7) and either (8) or (9) are the full statement. These were numerically integrated for the inviscid asymptote, for the uncoupled circumstance, and in the fully coupled form as discussed below.

PROCEDURE

Inviscid solution

The solution of the inviscid part of the momentum equation is first found for various Prandtl numbers. These results are important since they indicate the asymptotic behavior of both the coupled and uncoupled modes as αG becomes large.

The inviscid equation is obtained from equation (5), with the appropriate simplification. Considering a large value for αG, the left term of the equation may be neglected compared to the terms on the right. The resulting second order differential equation is:

$$\phi'' = \left[\alpha^2 - \frac{F'''}{F' - \beta/\alpha}\right]\phi. \qquad (10)$$

The boundary conditions necessary for the integration of the second order differential equation are:

$$\phi(0) = 0, \quad \phi(\infty) \to 0 \quad \text{(Symmetric disturbances)} \qquad (11)$$

$$\phi'(0) = 0, \quad \phi(\infty) \to 0 \quad \text{(Asymmetric disturbances).}$$

Based on equation (10), Shen [7] has listed some general conditions which were obtained when amplified disturbances are assumed to exist. The proof of these results is not reproduced here. It is noticed, however, that since the plume clearly has an inflection point (see Gebhart *et al.* [6]) it follows that for this case, equation (10) has a positive real eigenvalue α^2. Subject to the boundary conditions in (11), it is found by direct numerical integration of equation (10) that, for $\sigma = 0{\cdot}7$,

$$\alpha = 1{\cdot}3847 \quad \text{(Asymmetric disturbances)}$$

and

$$\alpha = 0{\cdot}7088 \quad \text{(Symmetric disturbances).}$$

If the actual neutral stability curve is found at lower values of α, as is subsequently seen to be the case, the above two values indicate that the flow is much less stable for asymmetric disturbances. The lower branch of the neutral curve is also known to be asymptotic to $\alpha = 0$ as $G \to \infty$ since $\alpha = 0$ is the other inviscid asymptote. In the following calculations, only the asymmetric case is considered further.

Effect of Prandtl number on inviscid instability

It is not the object of this work to present a detailed study of the effect of the Prandtl number on plume stability. Nevertheless, it is convenient to know if an increase or decrease in the stability limit is to be expected by a variation of Prandtl number. Since the coupled and uncoupled solution merge toward the inviscid behavior at large G, these asymptotic values were computed for a wide range of Prandtl number. If the value of the asymptote increases the region of instability also increases.

Figures 2 and 3 indicate the Prandtl number dependence of this stability limit. The value of β apparently reaches a minimum at $\sigma = 1$, whereas α appears to approach an asymptotic minimum as $\sigma \to 0$.

Uncoupled solution, including viscous effects

For moderate values of αG, the simplifications made above cease to be valid and the viscous

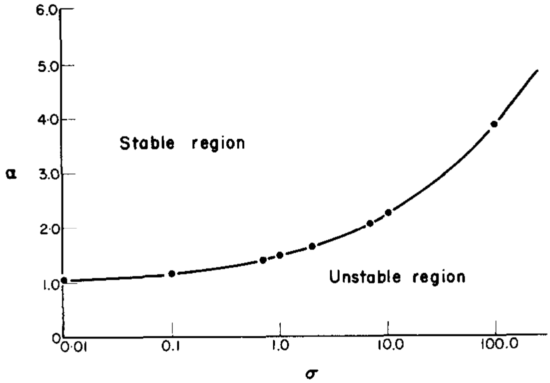

FIG. 2. Effect of Prandtl number on stability. Inviscid and uncoupled case. Asymmetric disturbances.

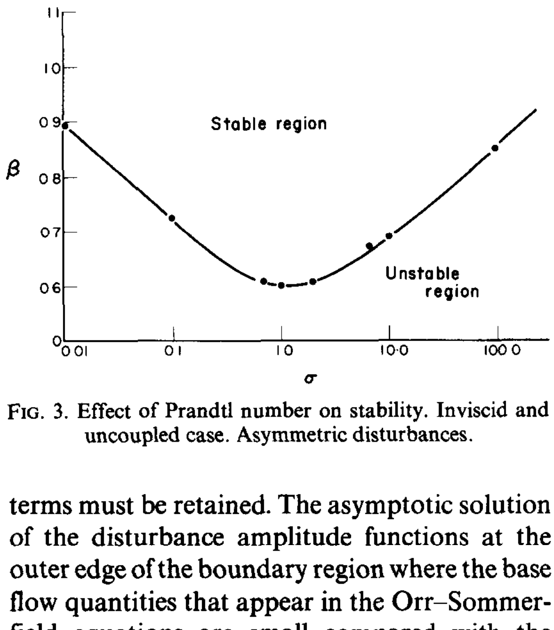

FIG. 3. Effect of Prandtl number on stability. Inviscid and uncoupled case. Asymmetric disturbances.

terms must be retained. The asymptotic solution of the disturbance amplitude functions at the outer edge of the boundary region where the base flow quantities that appear in the Orr–Sommerfield equations are small compared with the other terms is:

$$\phi = e^{-\alpha\eta} + B\,e^{-b\eta} \tag{12}$$

where $b = \sqrt{(\alpha^2 - i\beta G)}$, $b_{re} > 0$ and B a constant.

In accordance with the method of Dring and Gebhart [8] for flow over a vertical surface, the procedure used here is to integrate the viscous momentum disturbance equation from the edge of the boundary layer to the mid-plane of the plume, simultaneously satisfying boundary conditions at zero and the asymptotic solution at the edge of the boundary layer. The integration was started at the outer edge with the asymptotic solution and was continued toward the mid-plane. A system of eigenvalues and parameters was considered to be correct when the asymptotic expression provided a set of starting conditions at the outer edge which resulted in a satisfaction of the boundary conditions at $\eta = 0$.

There is no reason to believe *a priori* that this uncoupled solution gives a proper representation of the stability of the plume. It was found by previous investigators that in similar natural convection problems the coupling term is important and may not be neglected. It is, therefore, necessary to also determine the coupled solution and to estimate the importance of the coupling effect.

Coupled solution

The method that was employed in solving the system of coupled equations is somewhat different from that employed in the uncoupled case above. The asymptotic behavior (for large η) of (5) and (6) for the velocity and temperature disturbance amplitudes, analogous to (12) for the uncoupled case, was reported by Nachtsheim [9]. Dring and Gebhart, using this asymptotic behavior and following a procedure similar to the one described above for the uncoupled case, were able to solve the coupled equations for natural convection flow near a vertical plate. Since the method requires an interpolation in a six-dimensional space, it was found to converge very slowly to a solution in a stability plane.

Hieber [10] has shown that instead of employing the above procedure. it is more efficient to individually compute the three linearly independent integrals which decay exponentially for large η. Denoting these integrals by ϕ_1, ϕ_2, ϕ_3 (with associated temperature disturbances s_1, s_2, s_3), one has that, as $\eta \to \infty$,

$$\phi_1 \sim e^{-\alpha\eta}, \qquad \phi_2 \sim e^{-b\eta}, \qquad \phi_3 \sim e^{-d\eta}, \tag{13}$$

where b was defined by (12) and

$$d = \sqrt{(\alpha^2 - i\beta G\sigma)}. \tag{14}$$

Since the asymptotic behavior of the base flow temperature profile is

$$\Phi = C\exp(-2\cdot 4\sigma F_\infty \eta) \tag{15}$$

(where $F_\infty = \lim_{\eta\to\infty} F(\eta)$ and is equal to 0·931 for $\sigma = 0\cdot 7$) it follows from (6) and (13) that as $\eta \to \infty$,

$$\begin{aligned} s_1 &= C_1 \exp[-(\alpha + 2\cdot 4\sigma F_\infty)\eta] \\ s_2 &= C_2 \exp[-(b + 2\cdot 4\sigma F_\infty)\eta] \\ s_3 &= C_3 \exp(-d\eta) \end{aligned} \tag{16}$$

where C_1, C_2 and C_3 are appropriate constants.

Denoting the complete solution by

$$\phi = \phi_1 + A\phi_2 + B\phi_3 \tag{17}$$

one proceeds to determine ϕ_1, ϕ_2, ϕ_3 (and, concomitantly, s_1, s_2, s_3) by numerically integrating equations (5) and (6) from the edge of the plume to $\eta = 0$; in each case, the appropriate asymptotic behavior in (13) and (16) are employed as initial conditions. Having determined the three independent integrals at $\eta = 0$, one evaluates A and B by applying the two boundary conditions $\phi'(0) = 0 = \phi'''(0)$. The third boundary condition at the midplane, $s(0) = 0$, will in general not be satisfied. Supposing G and β to be fixed, one then iterates on α (the eigenvalue), repeating the process until the third boundary condition is also satisfied.

In effect, then, this method reduces the problem to finding a point in two-dimensional (α is complex, in general) rather than in six-dimensional space. It is this procedure which has been employed in obtaining the coupled results which are discussed below.

RESULTS

Eigenvalues and eigenfunctions are the computed results of this study. The eigenvalue information for the coupled and uncoupled case is shown in Figs. 4 and 5 in plots of α and β vs. G. For neutral stability β is equal to β_{re} and α is α_{re}.

The neutral curve for the uncoupled case was computed for Prandtl number 0·7, and small sections of curves were obtained for $\sigma = 2\cdot 0$, 6·7 and 10 as shown on Fig. 4. Also shown on Figs. 4 and 5 is the coupled curve for $\sigma = 0\cdot 7$. For $\sigma = 0\cdot 7$, the upper branch of the coupled and uncoupled neutral curves appear to converge and to approach the inviscid asymptote, as expected. This suggests that the upper branch does not go to zero as G increases, i.e. even as $G \to \infty$ there exist a certain unstable range of wavelengths and this range seems to be wider for asymmetric disturbances.

The "critical" Grashof number for the uncoupled mode is 10·34 for $\sigma = 0\cdot 7$. The results calculated allowing for coupling are shown in Figs. 4 and 5, it is seen that the coupling term is important at low G but that its effect decreases as the Grashof number is increased. The computation in the very low G region do not reveal the existence of a lower branch of the neutral curve and of the "critical" Grashof number. At extremely low values of G our boundary layer simplifications are no longer applicable and therefore, the relevance of this fact is not clear.

In Fig. 5, the curves are plotted in terms of β. Since physical frequencies of small disturbances introduced into the flow remain constant as they are convected by the base flow, one may calculate the β vs. G path in this stability plane. For a plume, these paths are defined by $\beta G^{-\frac{1}{3}} = C$. Several of such paths are shown on Fig. 5 for the following test conditions (air, atmospheric conditions, $Q = 58\cdot 6$ Btu/h ft, wire length = 6 in., wire dia. = 0·005 in.). At low frequency, the path enters deeply into the unstable region as G increases and such disturbances are strongly amplified. At higher frequencies, however, the paths traverse less of the amplified region and disturbances are probably less amplified. At a sufficiently high frequency the path does not penetrate the amplified region and disturbances are always damped.

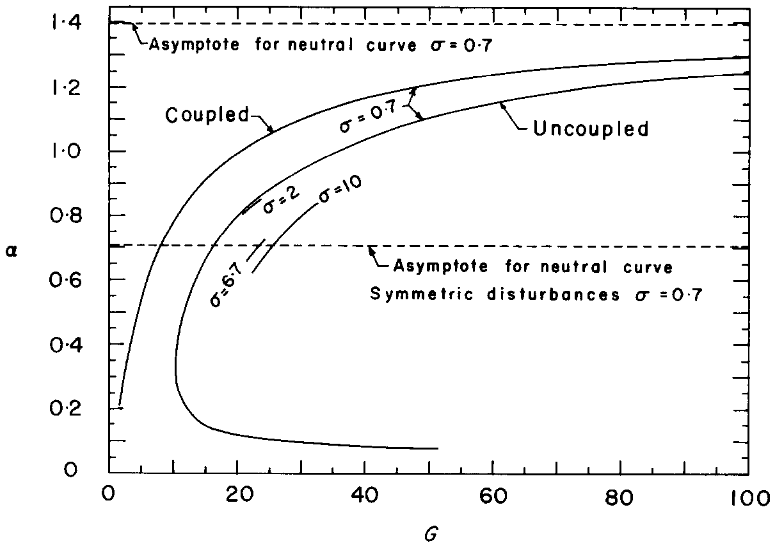

FIG. 4. Computed neutral stability curves. Coupled and uncoupled flow. Asymmetric disturbances.

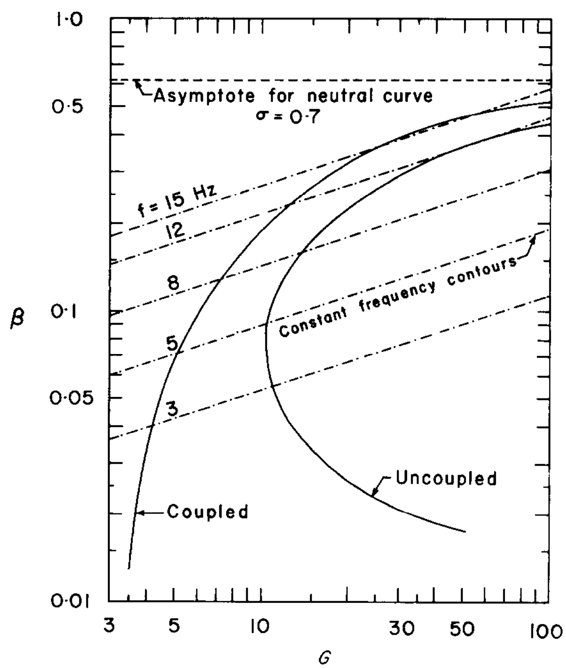

FIG. 5. Computed neutral stability curves. Coupled and uncoupled flow. Asymmetric disturbances. $\sigma = 0{\cdot}7$. –·–·–·– Constant frequency contours for air at test conditions.

Present results contrast with those obtained for natural convection flows over vertical plates. For such flows, constant frequency paths are defined by $\beta G^{\frac{1}{3}}$ constant. Therefore, disturbances of all frequencies cross the neutral curve and enter into the amplified region as they are convected downstream. One may say that such a flow eventually becomes unstable for all disturbances, regardless of frequency, if they are convected far enough downstream. In the present case, very high frequency disturbances appear to be always stable because they travel on paths that do not enter the unstable region. It can be shown that

$$\frac{\lambda}{x} = \frac{8\pi}{\alpha G} \tag{18}$$

where x is the distance from the source and α and G parameters computed along the neutral curve. Disturbances at the nose and along the low branch of the neutral curve are seen to have a very long wavelength, much longer than the distance to the source which generates the plume, thus decreasing the importance of this region.

EXPERIMENTAL OBSERVATIONS

The foregoing results predict the behavior of small disturbances as they are convected downstream in a plume rising from a line source of heat. In order to assess the reasonableness of these predictions, an experiment was performed. A 6 in. long wire of 0·005 in. dia. was electrically heated in air at atmospheric pressure and a 5 in. Mach–Zehnder interferometer was used to determine the temperature field above the source of heat. The light source was a Mercury vapor lamp with a green interference filter, the interferometer sensitivity being 7·25 degrees per fringe for a two-dimensional field, 6 in. wide. Adjustment was made to the infinite fringe, each fringe representing an isothermal contour.

An interferogram of the unperturbed plume in air was given by Gebhart *et al.* [6]. The interferogram clearly shows the extent of the thermal boundary region of the plume. The steadiness of the plume indicates the quiet surrounding in the test section. Since for a Prandtl number of 0·7 the velocity and the thermal boundary regions are of almost equal extent, the region seen is essentially the whole plume. The rectangular grid shown was introduced to check optical distortions of the system of lenses, and to serve as a frame of reference for distance measurements. The vertical distance between the lines is $\frac{1}{2}$ in. and the horizontal distance is $\frac{1}{4}$ in.

The study of stability was performed by introducing small controlled sinusoidal disturbances in the flow by means of a vibrating ribbon. The vibrator was a strip of metallic foil 0·005 in. thick and $\frac{1}{8}$ in. high. It was 7 in. wide and positioned horizontally in the mid-plane of the plume, above and parallel to the line source.

Figure 6 shows a sequence of plumes in air perturbed with controlled sinusoidal oscillation at different frequencies. Low frequency disturbances are strongly amplified and after a few oscillations the laminar base flow is completely transformed. As the frequency is increased, the amplification rate of the disturbances appears to be less, and a longer distance is apparently required to completely disrupt the flow. Disturbances of yet higher frequency were not observed downstream.

The observed plume behavior is consistent with the calculated results presented in Fig. 5. The dotted lines represent the paths of disturbances convected downstream at constant frequency. The numerical values of frequencies assigned to each line were computed for the test conditions.

Corresponding to each case on Fig. 6 a similar path on Fig. 5 can be drawn. Interferograms perturbed with frequencies higher than 7 Hz are not presented since the disturbances are of relative small amplitude and are difficult to be visually detected. A hot wire constant-temperature anemometer was used to follow higher frequencies disturbances. It was found that disturbances with frequencies higher than about 12 Hz are not detected downstream. The computed results for coupled disturbances (Fig. 5), predict that disturbances with frequencies higher than 15 Hz are stable, since they do not enter the unstable region. This discrepancy is not unreasonable since the introduced disturbance is not of perfect asymmetric form and does not necessarily become asymmetric during convection. Recall that the plume flow is appreciably more stable to symmetric disturbances.

It was not possible in this experiment to determine conditions of neutral stability. The unstable region extends to very low values of G and, for our test conditions in air, this is at very small x (around 0·1 in.). Therefore it was not possible to introduce disturbances at even smaller x, moreover boundary layer simplifications are no longer valid very near the heat source.

CONCLUSIONS

Previous work in the stability of natural convection flows near vertical surfaces has shown that buoyancy effects arising from temperature disturbances couple with velocity disturbances to promote instability. For an unbounded plume flow it is similarly found that the coupling effect is important, especially at low G, and that it may not be neglected.

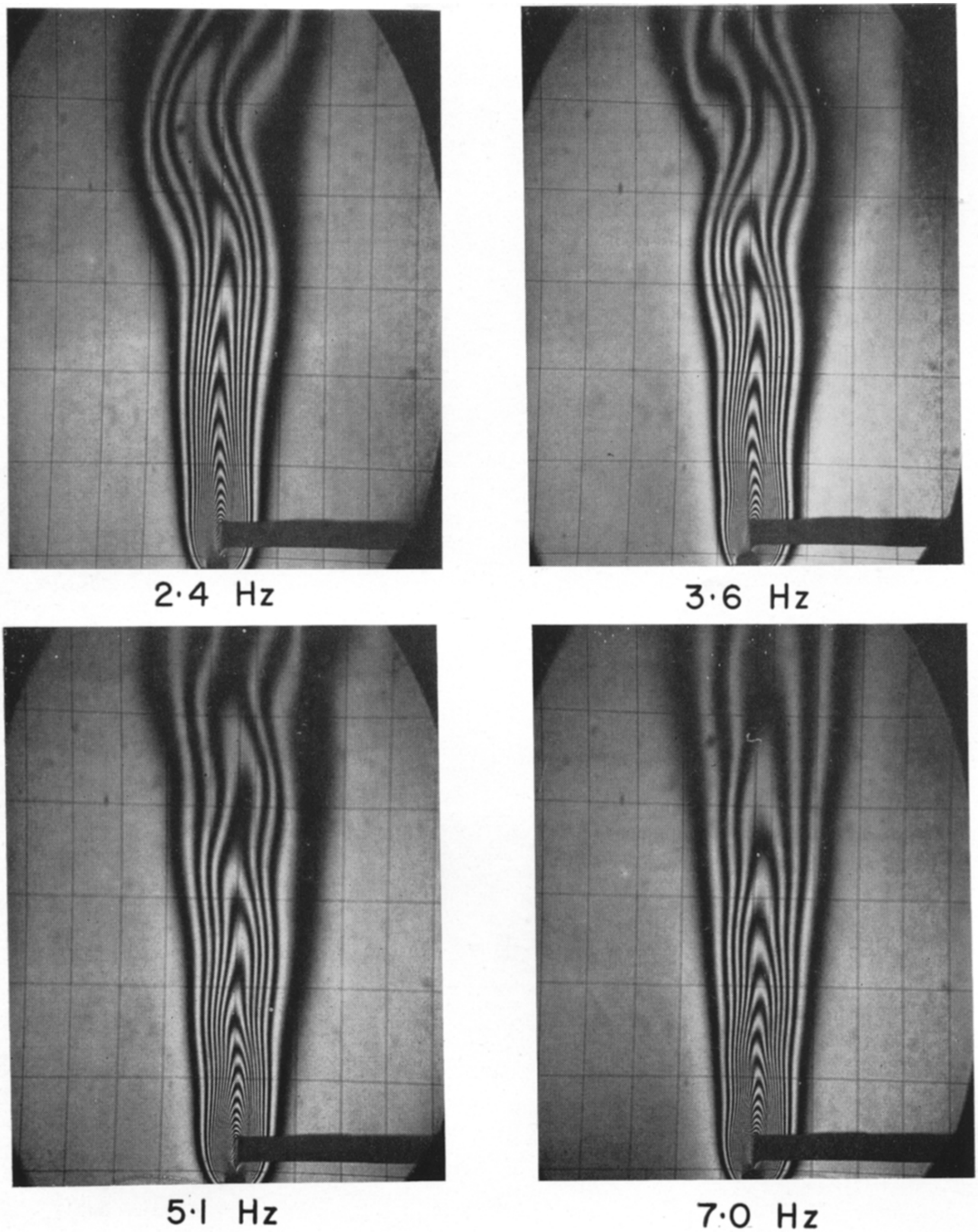

FIG. 6. Plumes perturbed with sinusoidal disturbances at several frequencies. Air at atmospheric conditions. Q = 58·6 Btu/h ft, wire length = 6 in., wire dia. = 0·005 in.

The experimental results obtained in the present work are not quantitative or conclusive, since no experimental determination of the neutral curve has been made. However, they do indicate that disturbances of low frequency are strongly amplified and that disturbances of sufficiently high frequency are damped as they move downstream.

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SUR LA STABILITÉ DE PANACHES LAMINAIRES: QUELQUES SOLUTIONS NUMÉRIQUES ET EXPÉRIENCES

Résumé—On a réalisé une étude de la stabilité hydrodynamique d'un panache laminaire s'élevant d'une source thermique linéaire à l'aide de la théorie des petites perturbations de Tollmien–Schlichting. Des solutions de fluides non visqueux de l'équation d'Orr–Sommerfeld sont obtenues pour des perturbations à la fois symétriques et dissymétriques et on a calculé l'effet du nombre de Prandtl sur la stabilité sans viscosité pour des perturbations dissymétriques. On trouve que l'écoulement de base est moins stable pour le mode dissymétrique. En plus toutes les équations de perturbation de la quantité de mouvement couplées ou non à l'équation d'énergie, sont numériquement intégrées avec les conditions limites appropriées pour les perturbations dissymétriques superposées à l'écoulement de base du panache symétrique. On obtient des courbes de stabilité neutre en fonction du nombre de Grashof.

En perturbant un panache dans l'air avec un ruban vibrant on a vérifié expérimentalement la prédominance du mode supposé dissymétrique des oscillations de l'écoulement. Un interféromètre Mach–Zehnder est utilisé afin d'observer comment les perturbations sont convectées en aval.

Les résultats expérimentaux démontrent que des perturbations à fréquence suffisamment hautes sont stables quand elles sont convectées en aval.

DIE STABILITÄT LAMINARER AUFTRIEBSSTRÖMUNGEN: EINIGE NUMERISCHE UND VERSUCHE

Zusammenfassung— Eine Untersuchung der hydrodynamischen Stabilität einer laminaren Auftriebsströmung, ausgelöst von einer horizontalen Linien-Wärmequelle, wurde unter Verwendung der Tollmein-Schlichting-Theorie für kleine Störungen durchgeführt.

Lösungen der Orr-Sommerfeld-Gleichung bei Vernachlässigung der Viskosität erhält man sowohl für symmetrische wie unsymmetrische Störungen, der Einfluss der Prandtl-Zahl auf die "nicht zähe Stabilität wurde für unsymmetrische Störungen berechnet. Es zeigt sich, dass die Grundströmung für den unsymmetrischen Modus zu wenig stabil ist. Die vollständige Störungs-Impuls-Gleichung, gekoppelt und entkoppelt mit der Energie-Gleichung, wurde mit geeigneten Randbedingungen durch Überlagerung unsymmetrischer Störungen für die symmetrische Auftriebsströmung numerisch integriert. Neutrale Stabilitätskurven erhält man als Funktion der Grashof-Zahl.

Das Vorherrschen des angenommenen unsymmetrischen Falls der Strömungsschwingung wurde experimentell durch Störung einer Auftriebsströmung in Luft mit einem Vibrationsband nachgewiesen. Für die Beobachtung der durch Konvektion aufwärts wandernden Störungen wurde ein Mach-Zehnder-Interferometer verwendet.

Die experimentellen Ergebnisse zeigen, dass genügend hochfrequente Störungen stabil sind, während sie stromabwärts wandern.

К УСТОЙЧИВОСТИ ЛАМИНАРНЫХ СТРУЕК: НЕКОТОРЫЕ ЧИСЛЕННЫЕ РЕШЕНИЯ И ЭКСПЕРИМЕНТЫ

Аннотация—Исследование гидродинамической устойчивости ламинарной струйки, исходящей из горизонтального линейного источника тепла, проводилось, используя теорию малых возмущений Толмиена-Шлихтинга. Неявные решения уравнения Орра-Зоммерфельда получены как для симметричных, так и для асимметричных возмущений, а влияние числа Прандтля на неявную устойчивость рассчитано для асимметричных возмущений. Найдено, что основной поток менее устойчив к асимметричным возмущениям. Кроме того были численно проинтегрированы уравнения количества движения развитых возмущений совместно с уравнением энергии и без него при граничных условиях, соответствующих асимметричным возмущениям, наложенным на симметричное основное течение струйки. Получены неинтегральные кривые устойчивости в зависимости от числа Грасгофа. Преобладание предполагаемой асимметричной формы колебаний течения было проверено экспериментально путем возмущения струйки в воздухе с помощью колеблющейся ленты. Для наблюдения возмущений по мере их передвижения вниз по потоку использовался интерферометр Маха-Цендера. Результаты экспериментов показывают, что достаточно большая частота возмущений устойчива при их перемещении вниз по потоку.